Constraining the Neutron Star Mass and Radius Relation

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Outline

- Neutron stars. Formation. Characteristics
- Formalism used in the study of neutron stars
- Mass-Radius diagram
- Conclusions
- Perspective

Neutron Star in Canada

NEUTRON STARS

- **Collapsed core of massive stars.**
  - They result from the supernova explosion of a massive star, combined with gravitational collapse, that compresses the core past the white dwarf star density to that of atomic nuclei.
- **Smallest and densest stars known to exist.**
- **Mass:** $M > 1.4\, M_{sun}$
- **Radius:** $r \approx 7 - 10\, km$
- **Density:** $\rho \approx 10^{14} \div 10^{15}\, g/cm^3$

SIZE AND NUMBER OF BARYONS IN A STAR

Limit of gravitational collapse: R=2M

Gravity packs nucleons up to their repulsive cores:

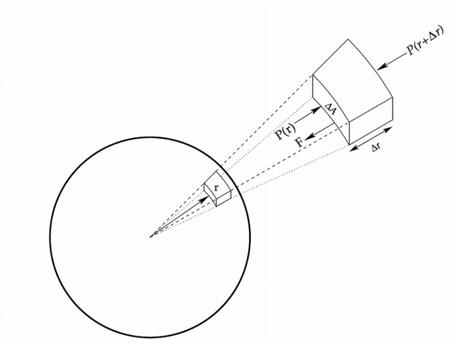
$$r_0 \approx 0.5 \times 10^{-13}\, cm$$

$$R \approx r_0 \cdot A^{1/3}, M \approx A \cdot m$$
$$m \approx 939\, MeV = 1.7 \times 10^{-24}\, g = 1.2 \times 10^{-52}\, cm$$
$$2M = r_0 \cdot A^{1/3}$$
$$2Am = r_0 \cdot A^{1/3}$$
$$A = 2.99 \cdot 10^{57}$$
$$R = 7.2\, km$$
$$M = \frac{R}{2} = 3.6\, km = 2.438\, M_{sun}$$

$$G = c = k_B = 1$$
$$c = 2.9979 \times 10^{10}\, cm/s$$
$$G = 6.672 \times 10^{-8}\, cm^3 g^{-1} s^{-2}$$
$$k = 1.3807 \times 10^{-16}\, erg/K, 1\, erg = 10^{-7}\, J$$

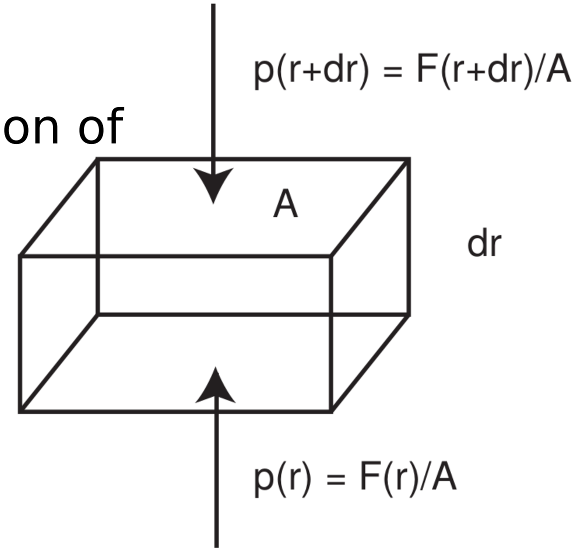
HYDROSTATIC EQUILLIBRIUM

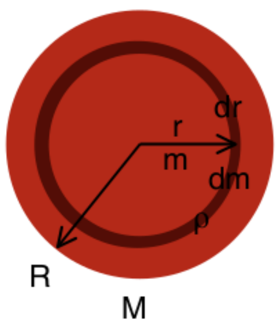
Newtonian Formulation



m is the mass interior to r, then conservation of mass implies that: $dm = 4\pi r^2 \rho\, dr$

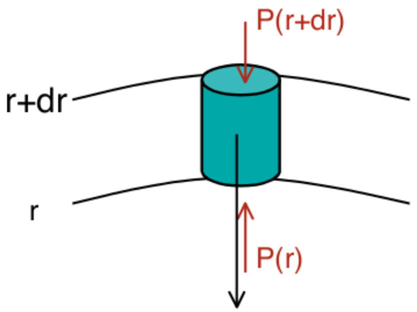
$$\frac{dm}{dr} = 4\pi r^2 \rho$$





Consider small cylindrical element between radius r and radius r + dr in the star.

Gravity(inward): $F_g = -G\frac{mM}{r^2}$



Pressure (net force due to difference in pressure between upper and lower faces): $F_p = P(r)dS - P(r+dr)dS =$

$$= P(r)dS - [P(r) + \frac{dP}{dr}dr]dS =$$

$$= -\frac{dP}{dr}drdS$$

HYDROSTATIC EQUILLIBRIUM

$$dm\,\ddot{r} = F_g + F_p = -G\frac{mM}{r^2} - \frac{dP}{dr}drdS$$

$$If\ the\ star\ is\ static:\ \Rightarrow \ddot{r} = 0$$

$$-G\frac{M\,\rho\,dr\,dS}{r^2} - \frac{dP}{dr}drdS = 0$$

Equation of hydrostatic equilibrium: $$\frac{dP}{dr} = -G\frac{M\rho}{r^2}$$

$$\epsilon = \rho\,c^2, \epsilon = energy\ density$$

$$\boldsymbol{\frac{dP}{dr} = -G\frac{M\,\epsilon}{r^2\,c^2}}$$

Equation of State

$$E_{tot}(n,x_i) = E_b(n,x_p) + E_{lep}(n,x_e,x_\mu)$$
$$E_b(n,x_p) = E_0(n) + S(n,x_p)$$
$$E_{lep}(n,x_e,x_\mu) = E_e(n,x_e) + E_\mu(n,x_\mu)$$

$$n = n_p + n_n \qquad x_i = \frac{n_i}{n}$$

- **For describing the baryonic part we need to consider the assymetry term:**

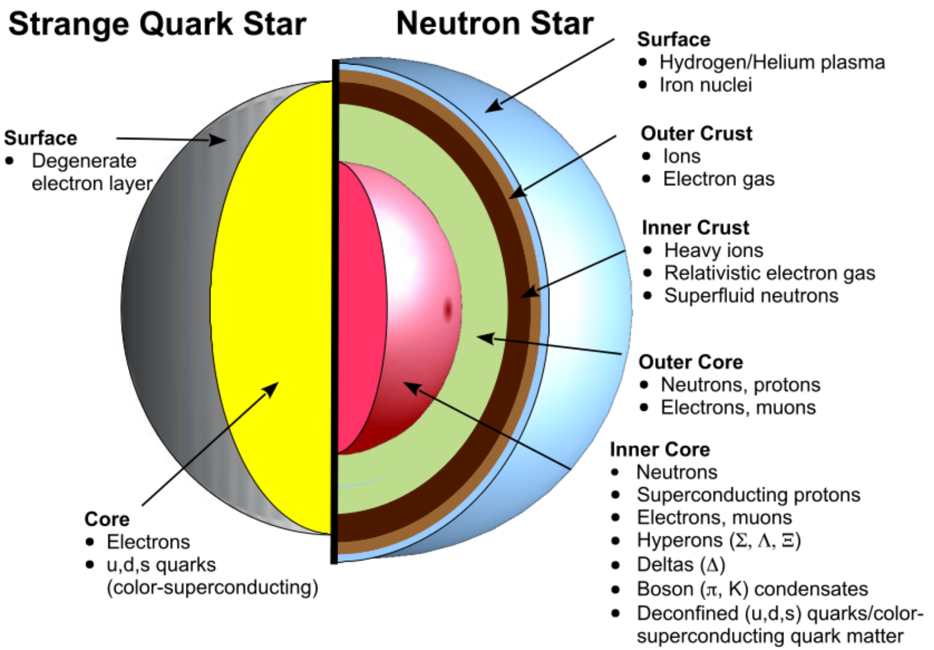
$$\alpha = \frac{n_n - n_p}{n_n + n_p} = 1 - 2x_p \qquad S(n,x_p) = (1-2x_p)^2\, E_s$$

- **For describing lepton's contribution:**

$$E_l(n,x_l) = \frac{1}{n}\frac{p_{F,l}^{\ 4}}{4\pi^2}\left[\sqrt{1+z_l^{\ 2}}\left(1+\frac{z_l^{\ 2}}{2}\right) - \frac{z_l^{\ 4}}{2}\arcsin\left(\frac{1}{z_l}\right)\right]$$

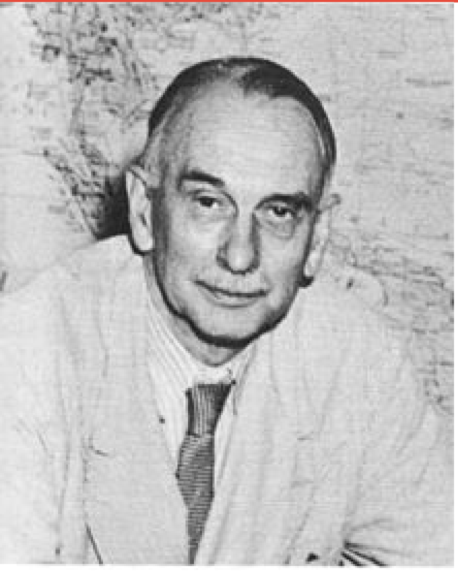
$$E_l(n,x_l)\Big|_{m_l=0} = \frac{1}{n}\frac{p_{F,l}^{\ 4}}{4\pi^2} = \frac{3}{4}(3\pi^2 n)^{1/3} x_l^{\ 4/3} \qquad z_l = m_l/p_{F,l}$$

- **Total pressure:** $$P(n) = n^2\left(\frac{\partial E_{tot}}{\partial n}\right)$$



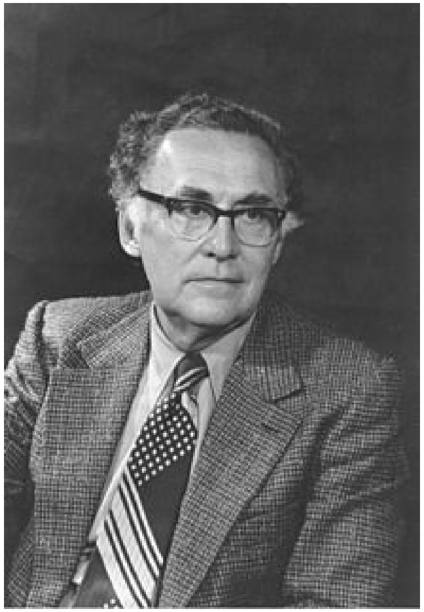
- Beta equilibrium: $\mu_n - \mu_p = \mu_e = \mu_\mu$
- Charge neutrality: $x_p = x_e + x_\mu$

Tolman-Oppenheimer-Volkoff equation + E.o.S.



$$\frac{dP}{dr} = -\frac{(\rho + p/c^2)\,G\,(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$



$p(\rho)$ or $p(\epsilon)$

- Initial condition: $p(\rho_c) = p_c$

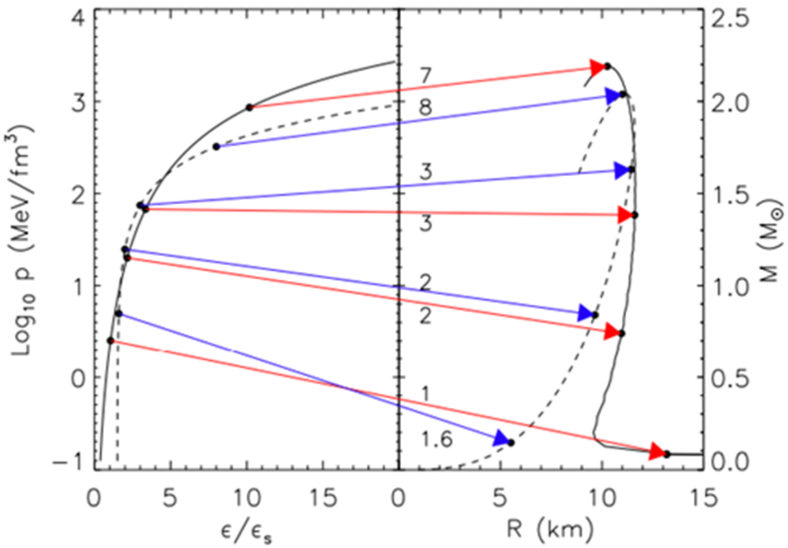
- Boundary conditions:

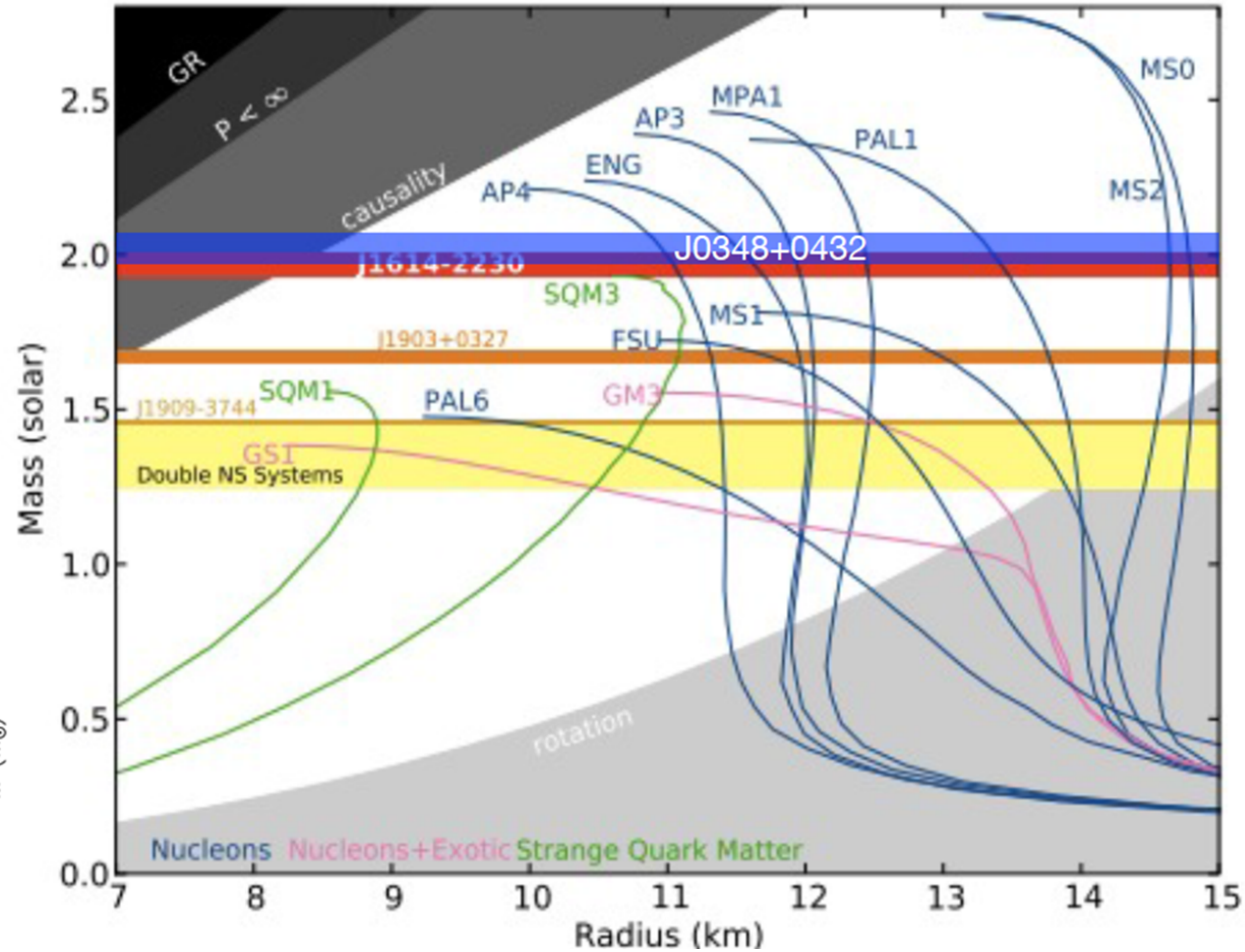
$$p(r=R) = 0$$
$$M(r=0) = 0$$

Tolman-Oppenheimer-Volkoff equation + E.o.S. - Mass-Radius Diagram

Constraints

- 2 M_{sun} observed
- Fastest rotation observed
- Causality
- Calculated maximum mass

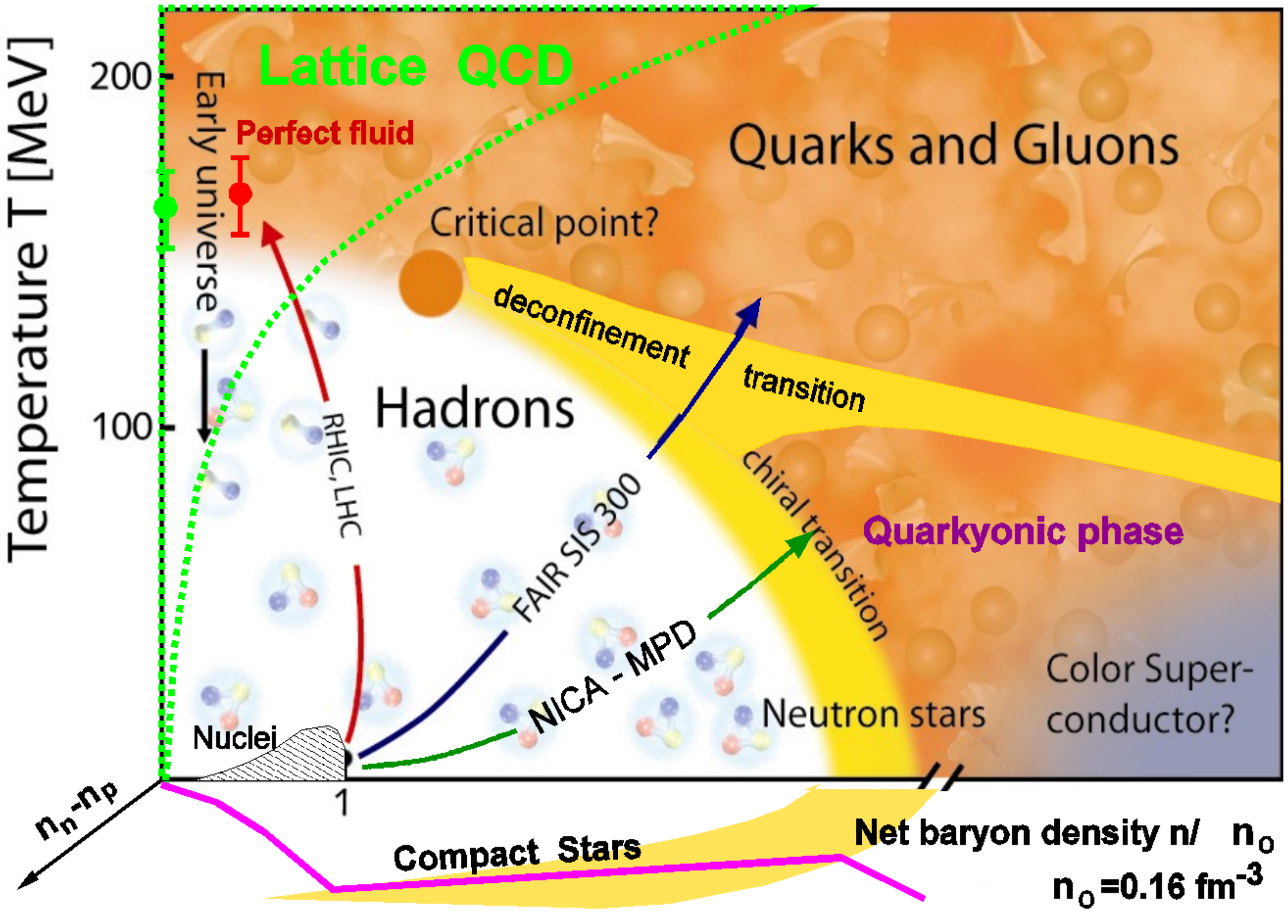




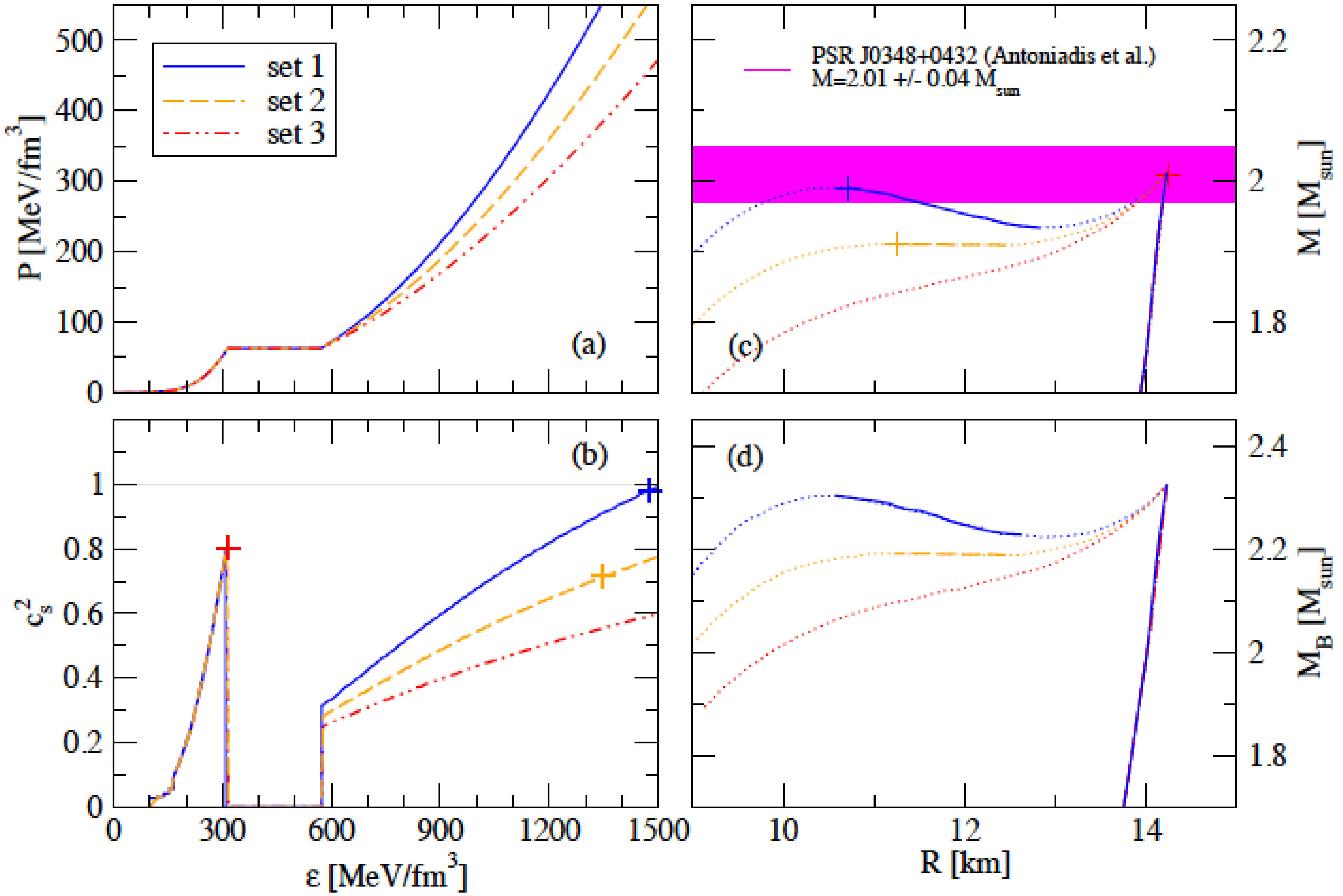
Phase Transitions

- Limit conditions:

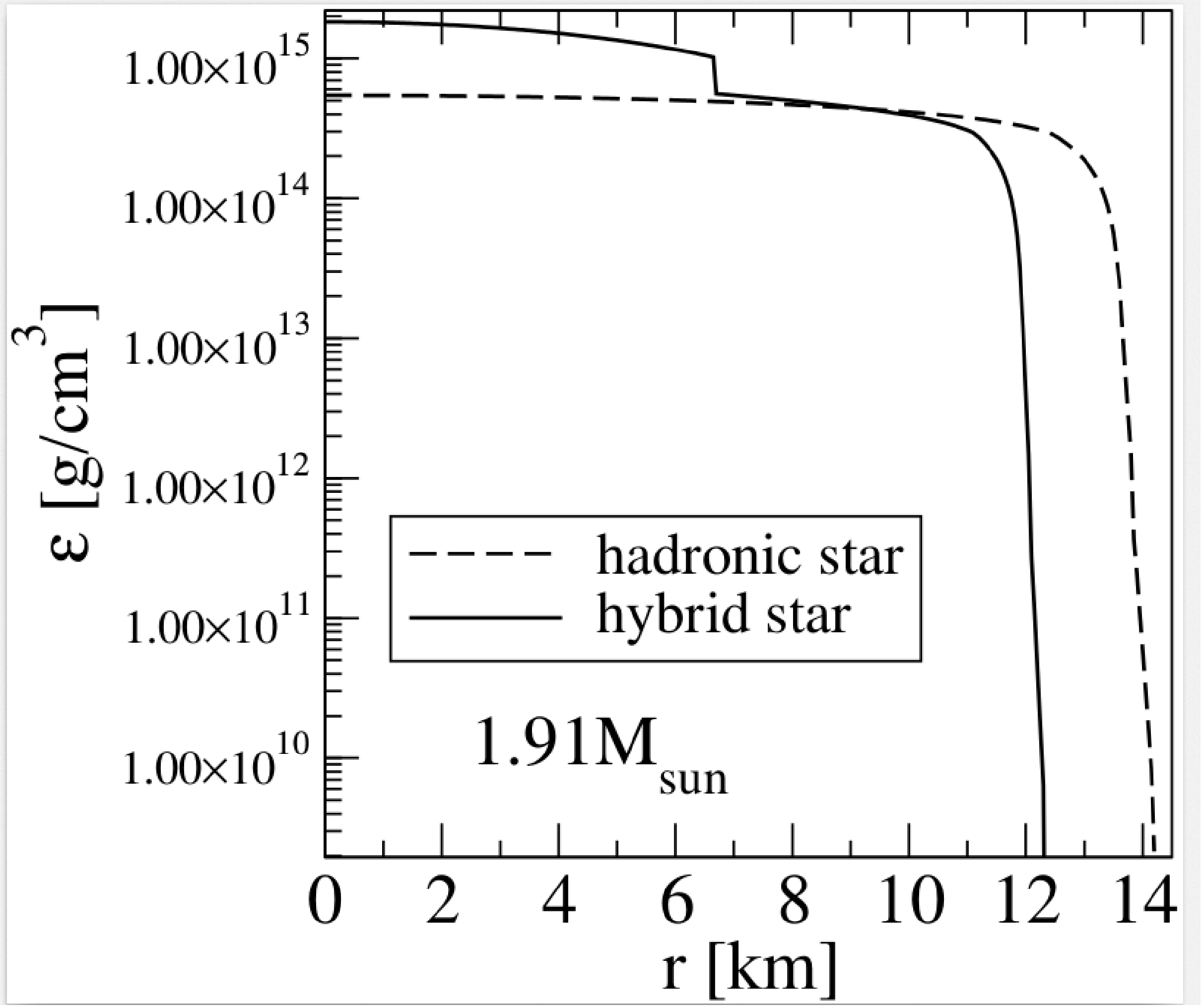
$$P^{I} = P^{II}; \quad \mu_n{}^{I} = \mu_n{}^{II}; \quad \mu_e{}^{I} = \mu_e{}^{II}$$



Phase Transitions



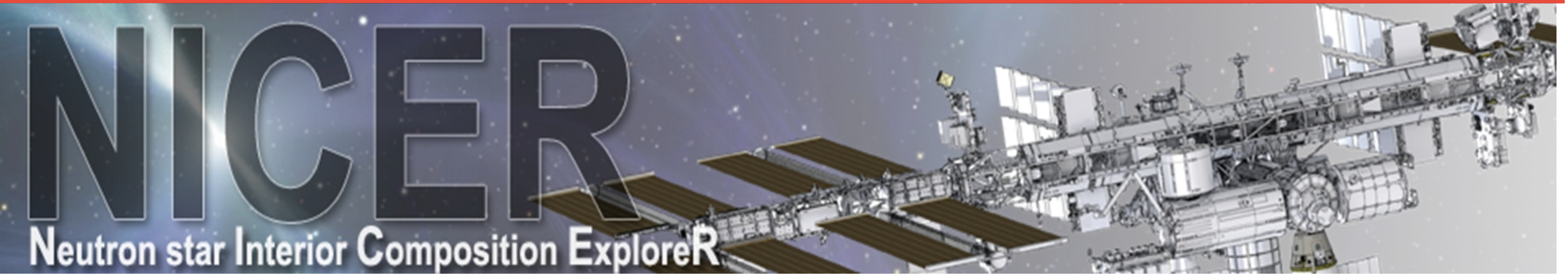
Internal Profiles

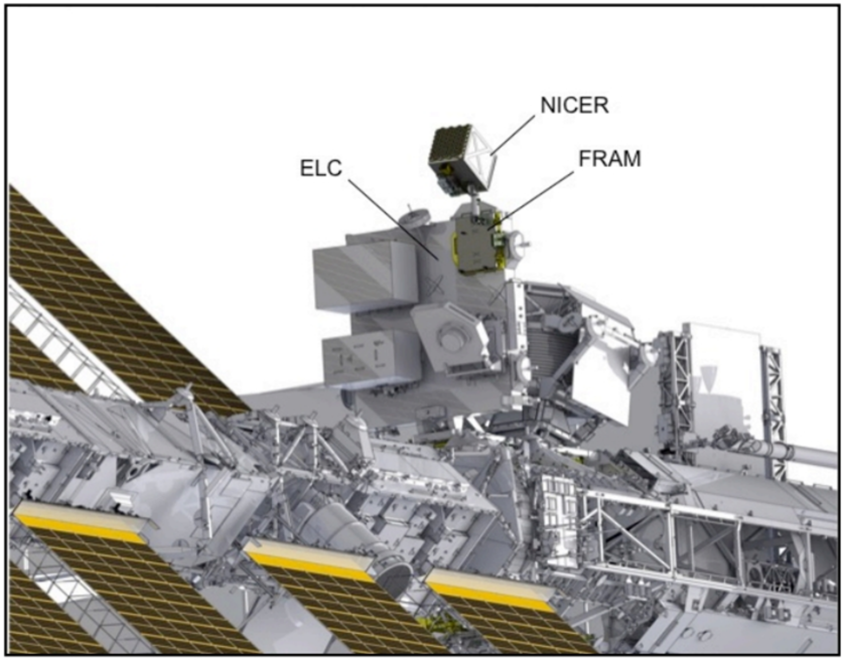


CONCLUSIONS

- **Phase transitions are evidence of the existence of different layers in the structure of neutron stars**
- **We used nuclear models to describe astronomic bodies and it is in concordance with observations**
- **In order to have a perfect model for describing neutron stars we need more experimental measurements and astrophysical observations**

THE FUTURE WILL BE „NICER“





- Mass and radius determinations with unprecedented precision.
- NICER’s results will discriminate between dozens of proposed “equation of state” theoretical models

References

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THANK YOU FOR ATTENTION!